MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Multiplication and Division

2.28 Combining division with addition and subtraction

Teacher guide | Year 6

Teaching point 1:

Division can be combined with addition and subtraction; when there are no brackets, division is completed before addition or subtraction; when there are brackets, the calculation within the brackets is completed first.

Teaching point 2:

When adding or subtracting division expressions that have a common divisor, the distributive law can be applied.

Overview of learning

In this segment children will:

- identify opportunities to group items and use division to complete calculations
- learn that when division is combined with addition or subtraction, the division must be completed first
- revisit the use of brackets to change the order of operations
- build on knowledge of the distributive law, introduced in segment *2.10 Connecting multiplication and division, and the distributive law* and expanded in segment *2.22 Combining multiplication with addition and subtraction*, extending it to apply to division problems.

This segment builds on segment *2.22*, where the distributive law was applied to multiplication problems with a common factor. In this segment children will apply the same principle to division problems:

- $a \div c + b \div c = (a + b) \div c$
- $a \div c - b \div c = (a - b) \div c$

Simple numbers are used so that children can focus on the structure of the calculations. Both quotitive and partitive division contexts are used throughout.

In *Teaching point 1* children explore context-based problems involving the addition or subtraction of two division expressions with different divisors (e.g. $10 \div 5 + 20 \div 4$ and $6 \div 2 - 12 \div 6$). Children are encouraged to identify groups and create their own division expressions from a variety of contexts. To begin with, children may need to draw a simple representation to help them visualise the problem, but children should be encouraged to work towards writing their own expression without the aid of a drawing.

Children are reminded of the role of brackets, applying this knowledge to the context of division to learn that if there are no brackets we know to do the division first; if there are brackets we do the calculation in the brackets first. By the end of this teaching point children will be confident in solving abstract problems involving the addition or subtraction of two division expressions with different divisors.

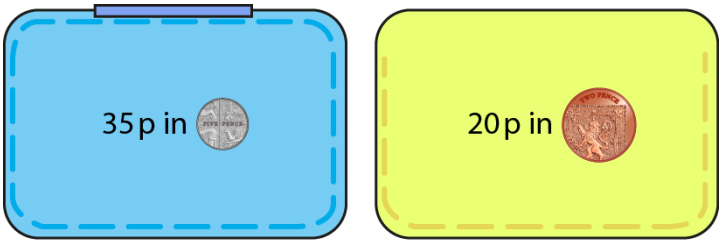
In *Teaching point 2* the distributive law is revisited in the context of division. Children will learn that when adding or subtracting two division expressions that have a common divisor, the distributive law can be applied. Children should be encouraged to look for the most efficient way to find solutions. This will prepare them for more complex calculations in the future.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

Division can be combined with addition and subtraction; when there are no brackets, division is completed before addition or subtraction; when there are brackets, the calculation within the brackets is completed first.

Steps in learning

	Guidance	Representations
1:1	This teaching point explores combining division with addition and subtraction. It follows the same structure as *Teaching point 1* in segment *2.22 Combining multiplication with addition and subtraction*, which looked at combining multiplication with addition and subtraction. This will prepare children for further learning on the distributive law, which was introduced in segment *2.10 Connecting multiplication and division, and the distributive law* and will be developed in *Teaching point 2*. To begin this teaching point, use context-based problems involving simple numbers to allow children to focus on the structure of the calculation. Start by showing them an example of two division calculations with different divisors, combined with addition, such as shown opposite. Children may be tempted to count the coins one by one, but encourage them to move away from a structure of repeated addition and to identify that they can use division to work out how many coins there are. First encourage children to identify that there are two different groups of coins (5 p coins and 2 p coins). Next we can use division to work out how many 5 p coins there are and how many 2 p coins there are, and then add the two quotients together to find the total. Work together to write an expression for the problem.	Quotitive division: *'Libby has 35 p in 5 p coins and Jen has 20 p in 2 p coins. How many coins do they have altogether?'*  Libby's coins Jen's coins 35 p ÷ 5 p + 20 p ÷ 2 p = 7 + 10 = 17  Libby's coins  Jen's coins • *'There are seventeen coins altogether.'*

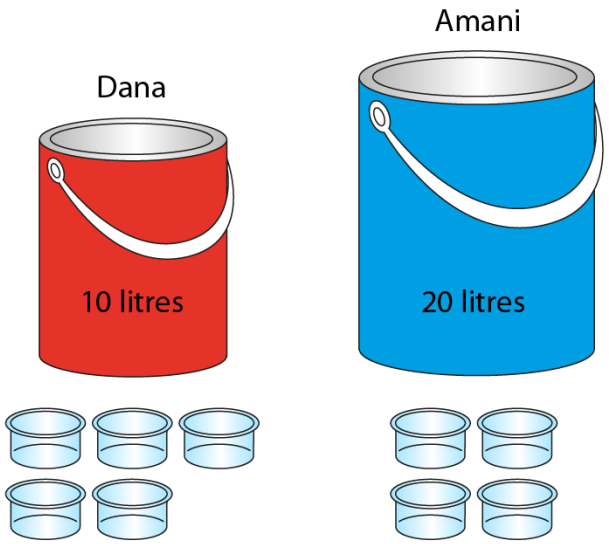
Repeat this context using coins to create similar problems, until children are confident with the structure.

Next move on to a context where the items are already organised into two groups and the individual items cannot be counted. To encourage children to visualise the problem, show them the written description before showing them the representation. Then ask children to write an expression to summarise the problem before moving on to calculate the answer.

Now provide children with practice solving problems that combine division and addition. Encourage children to draw a simple representation of the calculation until they are secure in their understanding. Use examples of both quotitive and partitive division. Children should build towards writing equations without needing to draw representations.

Partitive division:

- *'Dana has ten litres of red paint, which she pours into five pots. Amani has twenty litres of blue paint, which she pours into four pots. They give a pot of red paint and a pot of blue paint to Nathan. How much paint does Nathan have in total?'*



$10 \div 5 + 20 \div 4$

$= \quad 2 \quad + \quad 5$

$= 7$

- *'Nathan has seven litres of paint.'*

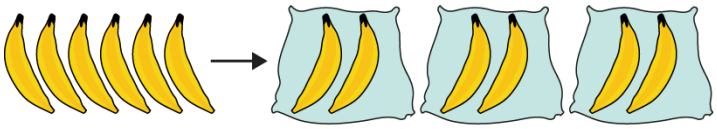
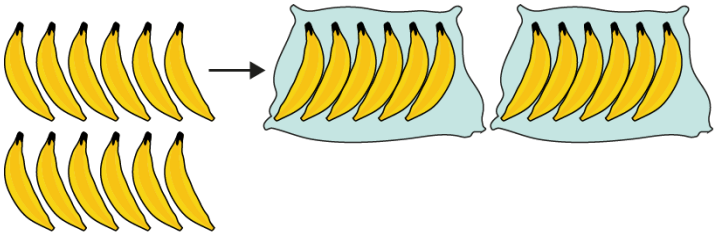
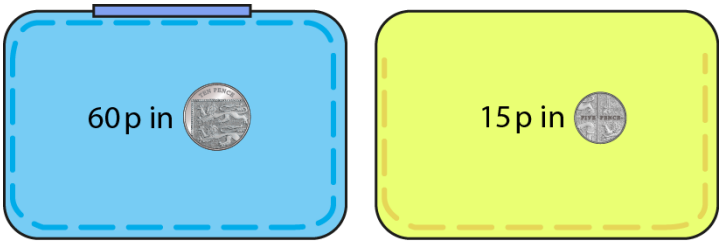
- *'Shop A sells 100 g of cereal shared into four small packs. Shop B sells 50 g of cereal shared into five small packs. If I buy a small pack from Shop A and a small pack from Shop B, how much cereal will I have altogether?'*

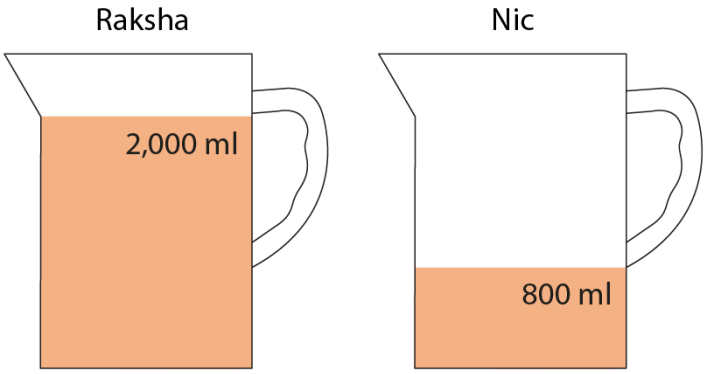
Shop A

100 g			
?	?	?	?

Shop B

50 g				
?	?	?	?	?

1:2	Move on to combining division with subtraction. Using examples such as the ones shown opposite, help children to identify that they can solve the two division problems and then subtract one quotient from the other. Representations can be used initially to support children in understanding the structure of the calculations. Encourage them to write down the steps of their working to build their confidence in writing equations. When secure in their understanding, children can solve problems by writing equations without the need for representations.	Quotitive division: • *'Leila puts six bananas into bags of two. James puts twelve bananas into bags of six. Who has more bags, and by how many?'* • *'Leila puts six bananas into bags of two.'*  • *'James puts twelve bananas into bags of six.'*  $6 \div 2 - 12 \div 6$ $= 3 - 2$ $= 1$ • *'Leila has one more bag than James.'* • *'Michael has 60 p in 10 p coins and Ting has 15 p in 5 p coins. Who has more coins, and by how many?'*  Michael's coins Ting's coins

		Partitive division: *'Raksha has 2,000 ml of water, which she pours into five glasses. Nic has 800 ml of water, which she pours into four glasses. Who has more water per glass, and by how much?'*  Raksha 2,000 ml ? ? ? ? ? Nic 800 ml ? ? ? ?
1:3	Next explore how an equation can sometimes have different solutions depending on the order in which the operations are carried out. Refer back to segment 2.22 *Combining multiplication with addition and subtraction, Teaching point 1*, where children were introduced to the order of operations in the context of multiplication. Start with a problem such as: *'Ali and Lily are solving the expression 12 + 16 ÷ 4. Who has solved it correctly? Explain your answer.'* • *'Ali says:'* 16 ÷ 4 = 4 4 + 12 = 16	

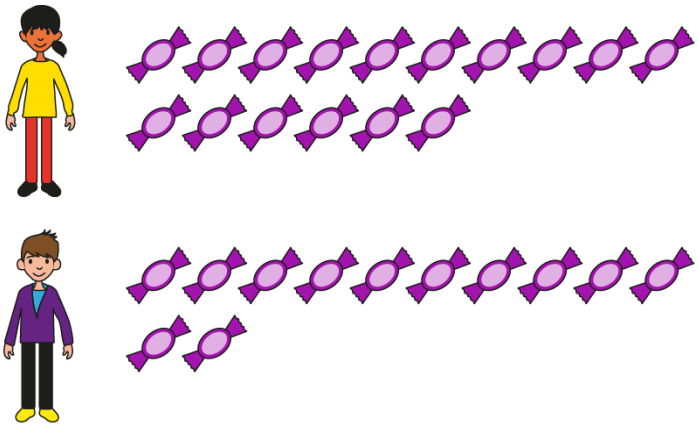
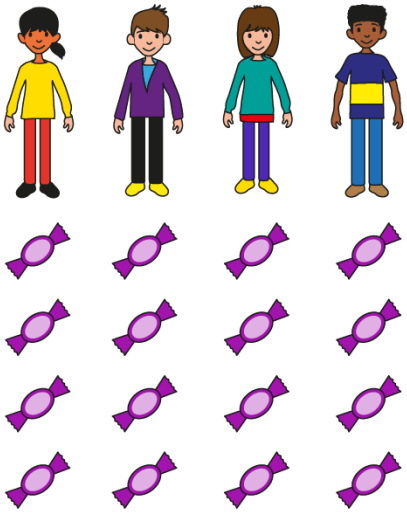
	• *'Lily says:'* 12 + 16 = 28 28 ÷ 4 = 7 Ask children to work through the two possible solutions and then ask them which is correct. Explain that division is normally carried out before addition or subtraction when there are no brackets, and therefore Ali is correct because he has completed the division first. Ask children how they could place brackets if they wanted to do the addition first: *(12 + 16) ÷ 4* Remind children that if there are no brackets we know to do the division first. If there are brackets we do the calculation in the brackets first. Repeat the exercise using similar expressions until children are confident with the order of operations and the use of brackets.	
1:4	Now extend the problem to include three operations: *'Now Ali and Lily are solving the expression 16 ÷ 4 + 12 ÷ 4. Who has solved it correctly? Explain your answer.'* • *'Ali says:'* 16 ÷ 4 = 4 12 ÷ 4 = 3 3 + 4 = 7 • *'Lily says:'* 16 ÷ 4 = 4 4 + 12 = 16 16 ÷ 4 = 4 Discuss how the two solutions were worked out, and come to the conclusion that Ali is correct again because he has carried out the division first. By the end of this step, children should be comfortable with the following generalisation: ***'When there are no brackets, division is completed before addition and subtraction.'***	

1:5	To finish this teaching point use missing number problems to deepen children's understanding of combining division with addition or subtraction. Remind children of the generalised statement in step *1:4* to ensure they are carrying out the calculations in the correct order: ***'When there are no brackets, division is completed before addition and subtraction.'***	*'Fill in the missing numbers.'* 35 ÷ 7 + 49 ÷ 7 = ☐ + ☐ ☐ ÷ 7 + ☐ ÷ 7 = 5 + 7 35 ÷ 7 + 40 ÷ 8 = ☐ ☐ ÷ 7 + ☐ ÷ 8 = 6 + 6 ☐ ÷ 6 – 30 ÷ 5 = 10 – 6 ☐ ÷ 6 – 24 ÷ 8 = 7 Dòng nǎo jīn: ☐ ÷ ☐ + ☐ ÷ ☐ = 10 + 6

Teaching point 2:

When adding or subtracting division expressions that have a common divisor, the distributive law can be applied.

Steps in learning

	Guidance	Representations
2:1	This teaching point explores division expressions with common divisors. This builds on segment *2.22 Combining multiplication with addition and subtraction, Teaching point 2*. We will work towards the distributive law, which children will also be familiar with from segment *2.10 Connecting multiplication and division, and the distributive law*. When applying the distributive law to division contexts, we learn that when the divisors are the same, we can first add the two dividends, then divide them by the common divisor. The distributive law for division can be represented algebraically as: $a \div c + b \div c = (a + b) \div c$ Children do not need to see the algebraic representation; this is for the teacher's information only. Show children an example such as the one opposite, using two division expressions with a common divisor. Before showing them the solutions, ask questions to encourage them to consider how they would approach the problem, for example: *'How could you work out how many sweets each child gets?'* Now work through Method 1, opposite, encouraging children to visualise the problem and to use the context to help them write an expression (e.g. $16 \div 4 + 12 \div 4$).	Partitive division: *'Ann has sixteen sweets and Brett has twelve sweets. They share the sweets between themselves and two other friends. How many does each person get?'*  Method 1:  • *'Ann's sweets are shared between four.'* $16 \div 4 = 4$

Ask children questions to deepen their understanding:

- *'What does the "16" represent?'* (Ann's sweets)
- *'What does the "12" represent?'* (Brett's sweets)
- *'What does the "4" represent?'* (the four children sharing the sweets)

Next ask children whether they can think of a different way to solve the problem. Work through Method 2, explaining that you could work out the total number of sweets first and then divide this by the number of children. Arrays such as the one opposite can be used to help children visualise the strategy. Ask children to write an expression using brackets, referring back to *Teaching point 1* if needed.

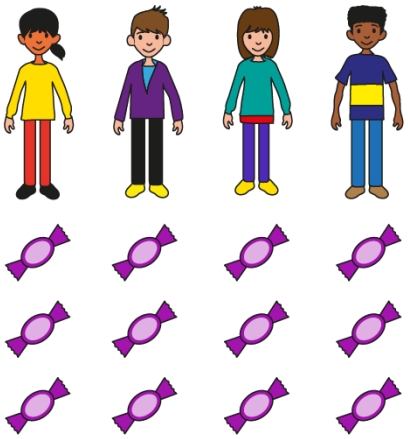
Continue to ask children questions:

- *'What does the "16" represent?'* (Ann's sweets)
- *'What does the "12" represent?'* (Brett's sweets)
- *'What does the "28" represent'* (all of the sweets)
- *'What does the "4" represent?'* (the four children sharing the sweets)

Now work through the coin example at the end of this step using the two different methods. Compare this to the coin example in step *1:1*, explaining that this problem is different because here both children have the same type of coin (5 p) and therefore there is a common divisor.

Start by writing the problem into an expression, using counters or real coins for support. After working through both methods ask children:

- *'Which method do you prefer and why?'*
- *'Is one more efficient than the other?'*



- *'Then Brett's sweets are shared between four.'*

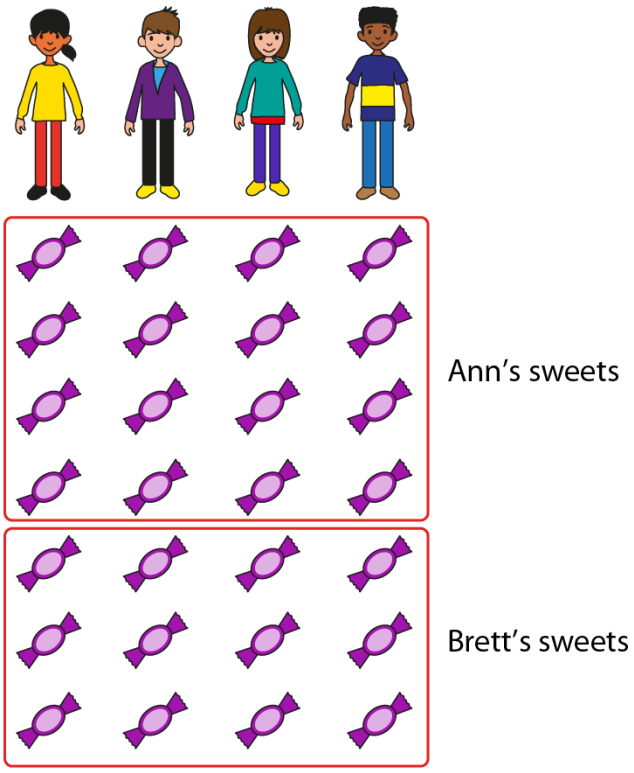
$12 \div 4 = 3$

$16 \div 4 + 12 \div 4$

$= \quad 4 \quad + \quad 3$

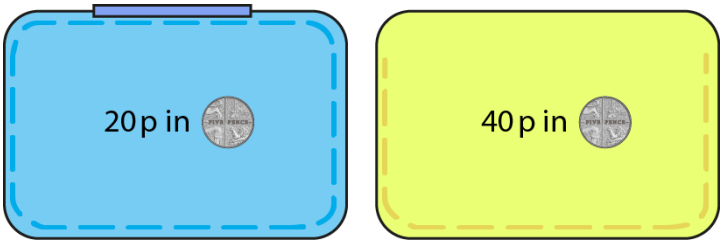
$= 7$

- *'Each child gets seven sweets.'*

	Guide children towards the answer that using the distributive law (Method 2) is more efficient. Summarise this step using the following generalisation: ***'When two dividends are divided by the same divisor, we can add the dividends first and then divide.'***	Method 2:  Ann's sweets Brett's sweets • *'There are twenty-eight sweets in total.'* $16 \div 4 + 12 \div 4$ $= (16 + 12) \div 4$ $= 28 \div 4$ $= 7$ • *'Each child gets seven sweets.'*

Quotitive division:

'Hanna has 20 p in 5 p coins and Adam has 40 p in 5 p coins. How many coins do they have altogether?'



Hanna's coins Adam's coins

Hanna's coins

Adam's coins

Method 1:

$20 \div 5 + 40 \div 5$

$= 4 + 8$

$= 12$

- *'There are twelve coins altogether.'*

Method 2:

$20 \div 5 + 40 \div 5$

$= (20 + 40) \div 5$

$= 60 \div 5$

$= 12$

- *'There are twelve coins altogether.'*

2:2	Provide children with practice using the distributive law to calculate the answer to a range of division questions with common divisors, for example: • *'A recipe requires 250 g of flour to make a pizza base for one person. Sean has 1 kg of flour and Simon has 2 kg of flour. How many pizza bases can they make?'* • *'There are eight tables in a classroom. John shares out thirty-two workbooks equally and Rachel shares out eight textbooks equally. How many books are there on each table?'* Encourage children to write an expression for each problem, and then to use the distributive law (Method 2 in step *2:1*) in their calculations. Refer to learning on brackets (*Teaching point 1*) and the distributive law (step *2:1*) if needed. Dòng nǎo jīn: *'In a classroom cupboard there are ten pairs of scissors, some glue sticks and twenty-five erasers. They are shared equally into five baskets for the children to use. If each basket has fifteen items, how many glue sticks were there?'*	

2:3

This step uses the same approach in order to solve problems that require subtraction rather than addition, and looks at problems in which children are required to find the difference between two expressions when one divisor is common to both expressions. This could be represented algebraically as:

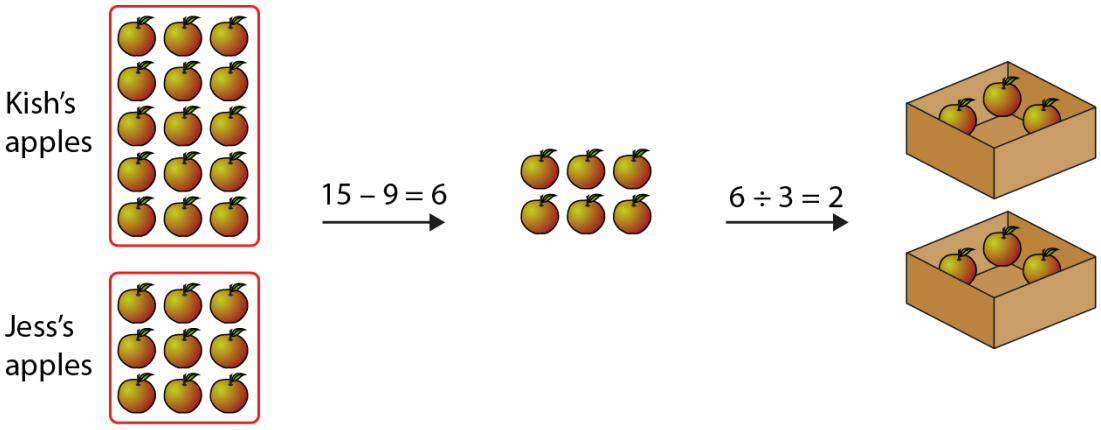
$a \div c - b \div c = (a - b) \div c$

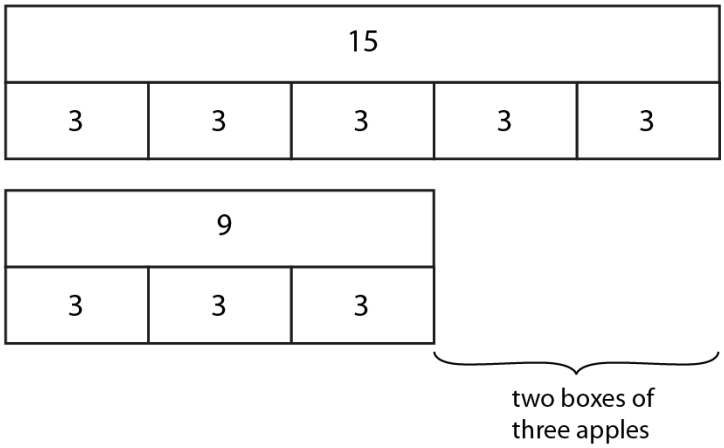
Provide children with problems such as those shown below. Use simple numbers to begin with, while children become familiar with the structure. Show them how the problems can be solved using the distributive law. Use representations to help children visualise the problems.

Summarise this step using the following generalisation: ***'When two dividends are divided by the same divisor, we can subtract the dividends first and then divide.'***

Quotitive division:

- *'Kish puts fifteen apples into boxes of three. Jess puts nine apples into boxes of three. Who has more boxes, and by how many?'*





$15 \div 3 - 9 \div 3$

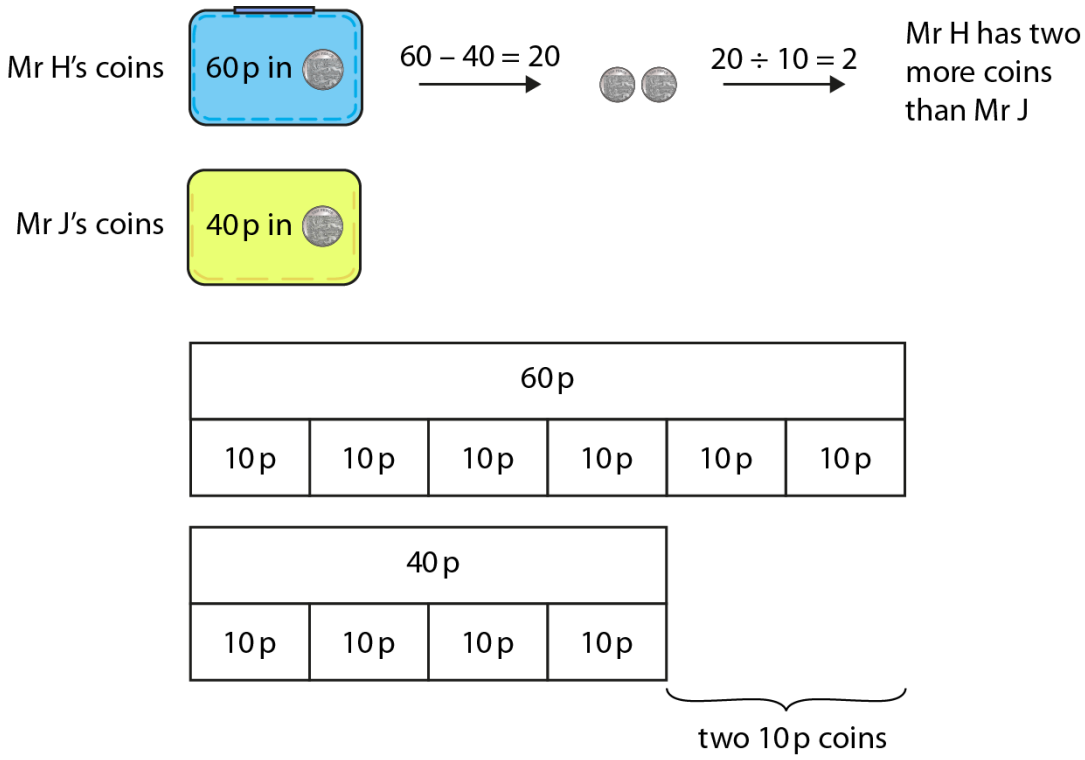
$= (15 - 9) \div 3$

$= 6 \div 3$

$= 2$

- *'Kish has two more boxes than Jess.'*

- *'Mr H has 60 p in 10 p coins and Mr J has 40 p in 10 p coins. Who has more coins, and by how many?'*



$$\begin{aligned} & 60 \div 10 - 40 \div 10 \\ = & (60 - 40) \div 10 \\ = & \ 20 \div 10 \\ = & \ 2 \end{aligned}$$

- *'Mr H has two more coins than Mr J.'*

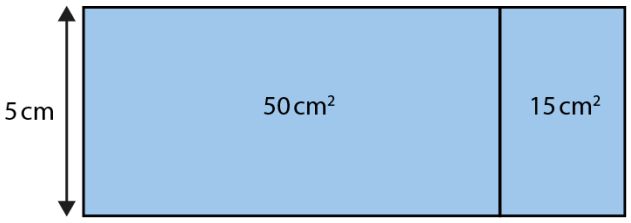
2:4

Provide children with varied practice applying the distributive law to contextual problems involving addition and subtraction. Include both quotitive and partitive division problems. For example:

- *'Class 1 and Class 2 have made some badges to sell at the school fair. Class 1 have made sixteen badges and Class 2 have made twenty badges. The badges are divided into four baskets to be sold. How many badges are in each basket?'*
- *'Tennis balls are stored in packs of three. Joe has twelve balls to put away. Milly has twenty-one balls to put away. How many packs are filled in total?'*

Dòng nǎo jīn:

'A rectangular piece of paper has been cut into two smaller rectangles. One of the smaller pieces of paper has an area of 50 cm² and the other has an area of 15 cm². If the width is 5 cm, what was the length of the piece of paper before it was cut?'



	• *'A classroom is given two new bookcases, filled with books on shelves. Each shelf fits twenty books. If one bookcase has one hundred and twenty books and the other bookcase has eighty books, how many shelves are full altogether?'* • *'Children from Year 5 and Year 6 do a sponsored run. They decide to donate the money equally to seven different charities. Year 6 raises £56 and Year 5 raises £84. How much money does each charity receive?'* • *'A company holds a charity dinner and raises £360 for one charity and £240 for the second charity. They keep $\frac{1}{12}$ of the money to cover the cost of the dinner. How much does the company keep?'*	
2:5	Finally provide children with intelligent practice using non-contextual expressions, referring back to steps *2:1–2:3* for support. Dòng nǎo jīn: *'True or false? Explain your answer.'* • *'Sarah says:'* $1000 \div 4 + 1000 \div 2 = 1000 \div 6$	*'Fill in the missing numbers and operators (+, – or ÷) using what you have learnt.'* $12 \div 4 + 16 \div 4 = (\square + 16) \div 4$ $10 \div 2 + 14 \div 2 = (10 + \square) \div 2$ $18 \div 3 - 12 \div 3 = (18 - 12) \div \square$ $24 \div 6 + 36 \div 6 = (\square + \square) \div \square$ $20 \div 5 + 20 \div 5 = (\square \bigcirc \square) \bigcirc \square$ $\square \div 7 - 14 \div 7 = (21 - 14) \div 7$ $20 \div 4 + \square \div 4 = (20 + 12) \div 4$ $21 \div \square - 12 \div \square = (21 - 12) \div 3$ $\square \div \square + \square \div \square = (12 + 12) \div 2$ $\square \bigcirc \square \bigcirc \square \bigcirc \square = (35 + 15) \div 5$